Tip Clearance Cavitation in Shrouded Underwater Propulsors

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The tip clearance flow over the blade ends of a turbomachine initiates two definite forms of cavitation termed "gap" and "tip vortex" cavitation. This paper attempts to characterize the flow pattern associated with various shaped gap configurations. The effects on the tip clearance flow, which are due to variation in tip clearance height, wall boundary layer, and blade tip loading of a rotating blade, are examined experimentally. Blade end configurations to obtain optimum gap and tip vortex cavitation resistance are suggested.

Nomenclature

C_L = lift coefficient
C_m = mass flow coefficient
C_p = standard pressure coefficient
C_p^* = maximum negative pressure coefficient at gap inlet
c = blade chord
E = modulus of elasticity
E_L = blade end coefficient as defined in Eq. (9)
h = clearance height
I = moment of inertia
K_l = loss coefficient
t = blade thickness
P = static pressure
P_c = mean static pressure at constricted area
P_m = static pressure at vortex core
P_o = static pressure on suction surface of blade end uneffected by gap flow
P_v = vapor pressure of fluid
P_t = total pressure along streamline
P_∞ = freestream static pressure uneffected by gap inlet flow
Q = flow per unit depth
$\bar{t}$ = average blade thickness
U = belt velocity
W = relative velocity
W_t = tip clearance flow velocity normal to blade chord
V_c = average velocity in constricted area
V_E = average velocity of gap exit
V_∞ = freestream velocity uneffected by belt motion
α = contraction coefficient
δ^* = momentum thickness of boundary layer
μ = absolute viscosity
ν = kinematic viscosity
ρ = mass density
σ = cavitation index as defined in Eq. (6)
Γ = vortex sheet strength

Introduction

THE phenomenon of cavitation has assumed increased importance in the design and operation of high-speed turbomachinery such as turbines, pumps, and shrouded propulsors on both underwater vehicles and surface vessels. One form of cavitation which has been observed quite commonly in axial flow machines is that which occurs in the tip clearance of the rotating blades. In order to prevent physical contact between a rotating member, such as a rotor blade and a guide wall, it is required that some finite clearance be maintained between the two, as shown in Fig. 1. It is in this area that cavitation quite often occurs before the inception of any other forms of blade cavitation. The objective of this investigation is to study the mechanics of tip clearance flow in turbomachinery. The specific area of study shall be concerned with cavitation that occurs in the gap or tip clearance region.

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Description of Tip Clearance Flow

The flow that passes through the clearance area between a blade end and a guide wall has been named the tip clearance flow. The tip clearance flow originates from the pressure difference across the blade tip section and the relative motion between the blade end and the adjacent guide wall. The tip clearance flow, as it passes through the tip clearance or gap area, attains a direction transverse to the main flow between the blades, and the subsequent interaction of these two flows creates disturbances in the blade passage and a flow pattern that deviates from that assumed to exist by usual design methods. The interaction of the tip clearance flow and the main flow has prompted many investigations, and Refs. 1–7 list some of the major studies.

Cavitation in turbomachinery may be characterized as a local reduction of static pressure to the vapor pressure of the liquid, the formation of a cavity within the flowing liquid, rapid pitting and the destruction of these parts of the machine in contact with the flowing fluid, losses in the efficiency of the machine, and accompanying noise and vibration. The "kind of cavitation" defines a characteristic form of cavitation by its location relative to some part of the machine. The following kinds of cavitation are defined here because of their frequent reference throughout this report:

1) "Gap cavitation" occurs on the blade tip in the tip clearance area between the blade end and the casing or shroud wall. This form of cavitation, as illustrated in Fig. 2, is the primary concern of this report.

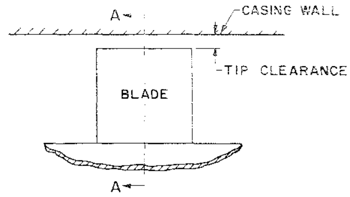
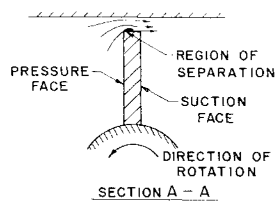


Fig. 1 Blade with tip clearance.



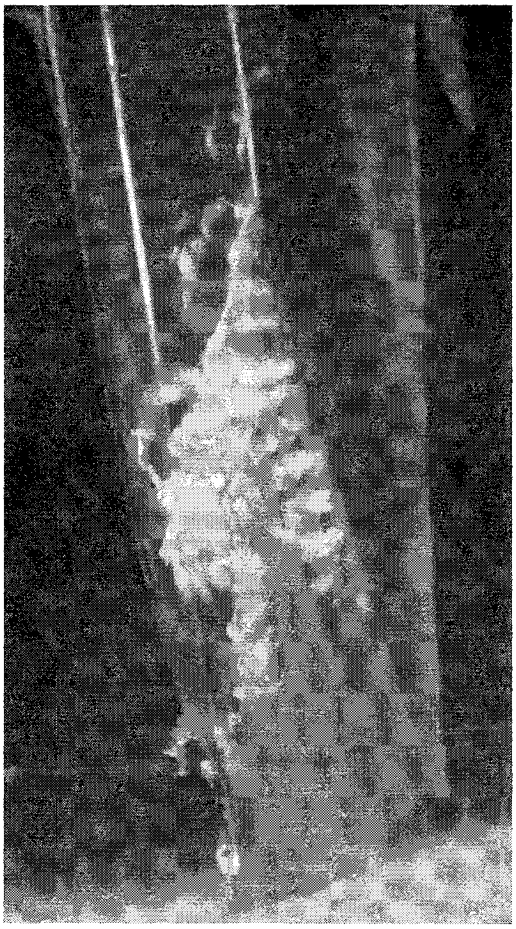


Fig. 2 Gap cavitation in tip clearance area.

2) "Tip vortex cavitation" originates from the interaction of the tip clearance flow and the main flow in the blade passage. The tip clearance flow rolls up into a vortex with subsequent cavitation in the vortex core. Cavitation of this form occurs initially as a small "feather" of cavitation attached to the leading edge tip of the blade, and as the local pressure is further reduced, the feather grows into a distinct cavity which extends from the leading edge of the blade and diagonally across the blade passage. This form of cavitation is illustrated in Fig. 3.

Fig. 3 Tip vortex cavitation.

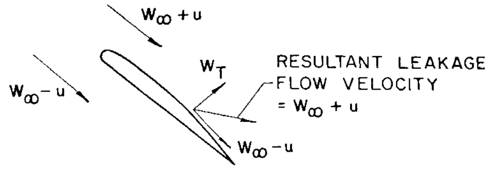


Fig. 4 Flow through a cascade of blades.

The "gap" and "tip vortex" forms of cavitation just discussed can be attributed directly to the tip clearance flow, and a fundamental analysis of these two kinds of cavitation is in order before a comprehensive evaluation of the experimental data can be achieved.

The "tip vortex" form of blade tip cavitation will be considered first, remembering that this kind of cavitation originates because of the leakage flow passing through the tip clearance and its subsequent interaction with the main flow in the blade passage on the suction side of the blade. The following analysis serves to exemplify this action while neglecting viscous effects and the associated skewed boundary layer passing through the tip clearance. A similar analysis was made by Rains.[6]

Referring to the blade cascade of Fig. 4, the fluid has a relative velocity (W_1) parallel to the pressure face of the blade. In general, the pressure gradient along the blade surface usually is small as compared to that across the blade end; it will be assumed that the velocity (W_1) is a component of the resultant velocity (W_2) of the leakage flow on the suction side of the blade.

Assuming that the viscous effects are small [therefore permitting the use of Bernoulli's equation to determine the magnitude of (W_T)], we have

$$P_1 + \rho(W_1^2/2) = P_2 + \rho/2(W_1^2 + W_T^2) = P_\infty + \rho(W_\infty^2/2) \quad (1)$$

Utilizing a simplified one-dimensional theory, a model can be developed representing the development of the vortex sheet produced by the interaction of the tip clearance flow and the through flow on the suction face of the blade. Assuming a perfect fluid, and using a perturbation approach where (u) equals some perturbation velocity, the model depicted in Fig. 5 results. Again, by Bernoulli's equation, we have

$$\frac{P_1 - P_2}{\rho(W_\infty^2/2)} = \frac{4u}{W_\infty} = C_L = \left(\frac{W_T}{W_\infty}\right)^2 = \Delta C_P \quad (2)$$

The resultant leakage flow velocity must equal ($W_\infty + u$) if the flow through the gap experiences no loss in energy. Therefore,

$$[(W_\infty - u)^2 + W_T^2]^{1/2} = W_\infty + u \quad (3)$$

The angle α, which the leakage flow makes with the blade chord, is

$$\alpha = \tan^{-1}\frac{W_T}{W_\infty - u} = \frac{C_L^{1/2}}{1 - C_L/4} = \frac{4C_L^{1/2}}{4 - C_L} \quad (4)$$

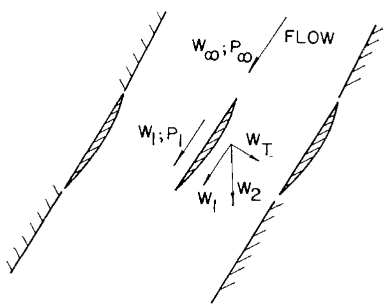
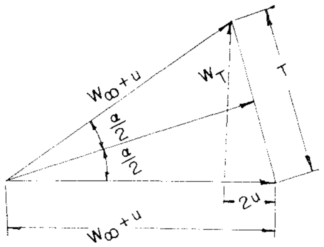


Fig. 5 Flow over a blade end

Fig. 6 Velocity diagram associated with tip clearance flow and blade passage flow.



The combination of the tip leakage flow and the through flow on the suction side of the blade gives the velocity diagram at the tip clearance exit, as shown in Fig. 6. The interaction of these two flows produces a vortex sheet on which strength per unit length is the vector difference between the tip leakage velocity vector and the through flow velocity vector. This vortex sheet is inherently unstable and will tend to roll up into a vortex. It is in the core of this vortex where cavitation eventually occurs. It might be assumed intuitively that, the stronger the strength of this vortex sheet, the greater is the tendency for tip vortex cavitation to occur. The preceding assumption similarly indicates that, the greater the mass flow through a given tip clearance height (h), the greater is the tendency for tip vortex cavitation. The existence of a vortex sheet as just described is indicated by photographs in Ref. 6 where it can be seen that local pressure has been reduced to a point where the entire vortex sheet is cavitating.

A vortex sheet is a thin layer of fluid across which the velocity is discontinuous. The strength of such a sheet per unit length (γ) is the product of the magnitude of the vorticity $/\delta/$ and the area over which this vorticity acts. In the case shown in Fig. 6, the sheet strength is $\gamma = T$, where

$$T = 2u/\sin(\alpha/2) \qquad \sin(\alpha/2) = 2u/(4u^2 + W_T^2)]^{1/2}$$

Therefore,

$$T = (4u^2 + W_T^2)^{1/2} \tag{5}$$

The preceding relation indicates that the sheet strength is a function of the velocity of the tip clearance flow as it issues from the gap. It suggests, in considering various gap configurations, that the tendency for the existence of tip vortex cavitation would be greater for a convergent gap than for a divergent gap. Justification for this can be based on the theory that, for a given pressure drop across a blade end, the gap exit velocity will be greater for the convergent gap, thereby providing a vortex sheet of high strength.

"Gap cavitation," as stated previously, occurs in the tip clearance area between the blade end and the casing wall. If a meridional cut is passed through a blade and the adjacent casing wall, a sectional view of the tip clearance area is seen. For an axial-flow machine of cylindrical cross section, the tip clearance section might be as shown in Fig. 1. For the case under consideration, the clearance height (h) is uniform and the tip-leakage flow approaching the clearance area from the pressure face of the blade must pass around the corner of the blade. To accomplish this, the velocity must ideally tend to infinity at the corner; but in attempting to achieve this, the flow will separate from the blade end. To analyze the flow, a two-dimensional potential solution, utilizing the Schwartz-Christoffel freestreamline method, might be used for predicting the pressure distribution on the casing wall along the length of the tap. Of course, this is an idealized flow pattern, which assumes that the separated region extends the length of the top clearance. This is not true in the actual case where the flow actually reattaches to the blade.

Shalnev[7] noted that cavitation originated in the separated region, and by considering a uniform tip clearance and the velocity distribution depicted in Fig. 7, he derived the following relation for cavitation index:

$$\sigma = \frac{P_\infty - P_v}{\rho(V_\infty^2/2)} = \frac{V^2}{\alpha^2 V_\infty^2}(2 + K_1) - 1 \tag{6}$$

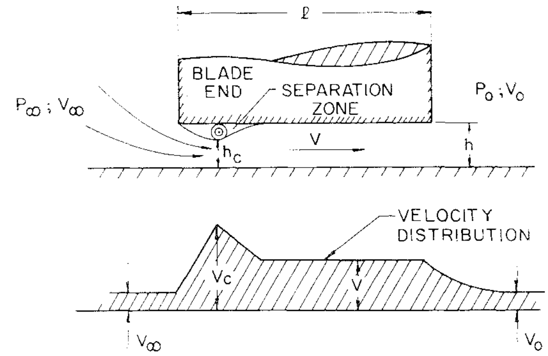


Fig. 7 Schematic of flow and velocity distribution through tip clearance.

This relation provides a theoretical cavitation index where $\alpha = h_c/h$ and K_1 = inlet loss coefficient. As indicated in Ref. 7, the coefficient α and K_1 for the case of a stationary blade are relatively constant for changes in the Reynolds number of the tip clearance flow. However, the effect of the relative motion of the blade end to the casing wall tends to change α and K_1 from those defined previously.

A method used in the past, to prevent separation on the blade end and the subsequent cavitation at the tip clearance inlet, has been accomplished by rounding the corner of the blade. The blade end would then be shaped as illustrated in Fig. 8. A "rule of thumb" for rounding the corner of the blade is to use a radius that is approximately equal to the gap height.[6] The effectiveness of such a criterion might be explained by comparing the contour obtained with the "rule of thumb" method and the freestreamline contour obtained by the potential solution for flow through a two-dimensional orifice. This is shown in Fig. 8 and it is seen that the contour for the "rule of thumb" method is more conservative than that required by the freestreamline, indicating that the fluid will not tend to separate from the blade end. The contour for the freestreamline solution was attained as outlined in Ref. 8.

Another gap configuration, which is likely to occur, is that form shown in Fig. 9. This configuration presents a divergent channel to the tip clearance flow as it proceeds from pressure face to suction face of blade. As will be shown in the section presenting experimental results, this gap configuration is most

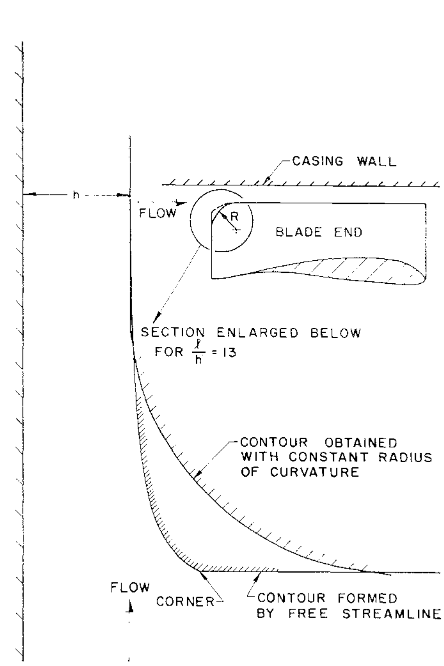


Fig. 8 Blade corner based on free streamline solution and that based on a constant radius of curvature.

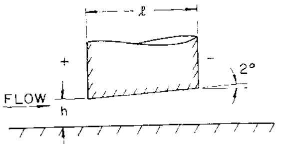


Fig. 9 Divergent gap.

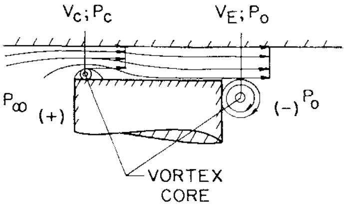


Fig. 11 Flow pattern associated with a uniform gap configuration.

undesirable because of its poor gap cavitation characteristics. However, this gap configuration can be obtained accidentally, even though a uniform gap was specified originally. Such a deviation from the designed configuration could be obtained when the casing wall has either a conical or spherical contour and the blades are tipped accordingly. Therefore, the clearance channel will consist of curved walls parallel to each other. Should the centers of the casing and the rotor deviate from their designed coaxial position, then a divergent tip clearance could result as shown. The divergent gap also could result from the deflection of the blade because of its hydrodynamic loading. The degree of divergence required to produce unfavorable gap cavitation performance, as indicated by tests, is very small and need only be as much as $\frac{3}{4}$ to 1°.

It is also possible to have a gap clearance which provides a convergent channel as the flow proceeds from blade pressure face to blade suction face. Figure 10 illustrates this type of tip clearance channel.

A qualitative analysis of the tip leakage flow and the dominating form of cavitation associated with various gap configurations can be accomplished by considering a simple one-dimensional model. In this simplified flow pattern two vortex systems will be assumed, one at the gap entrance and the second at the gap exit as illustrated in Fig. 11. The gap cavitation originates in the core of the vortex at the gap inlet, and it will be assumed that the tip vortex form of cavitation originates on the axis of rotation of the vortex at the gap exit. To obtain optimum cavitation performance for a particular gap configuration, it would be necessary to eliminate the occurrence of one form of cavitation before the other. This requires that, at the inception of cavitation, the pressure at the axis of rotation of each of the vortices be equal, assuming solid body rotation.[8] Referring to Fig. 11 for the condition of optimum cavitation performance,

$$P_c = P_v + \rho/2V_c^2 \qquad (7)$$

$$P_o = P_v + \rho/2V_E^2 \qquad (8)$$

In the case of a uniform gap with a sharp entrance corner, separation will occur and the velocity at the constriction (V_c) will be greater than the exit velocity (V_E). It can then be deduced by the conservation of energy for an ideal fluid that an optimum condition will not occur, and gap cavitation shall be the dominating form of cavitation for this gap configuration.

The previous analysis for various tip clearance configurations, although quite simplified, yields results that are in exact agreement with the test observations of Shalnev[7] in regard to the dominating form of cavitation for a particular gap configuration.

Experimental Model and Procedure

It was desired to have a test apparatus with which the mechanics of the flow within the gap area could be investigated. Since direct measurements of the flowfield in the clearance regions of a turbomachine would be quite difficult, it was apparent that a scaled-up model of the tip clearance flow must be obtained. To scale the gap height and blade end dimensions and still maintain a realistic flow pattern, it is also necessary to provide a boundary layer profile entering the gap which has been scaled by similar proportions. Figure 12 schematically illustrates the test apparatus used in this investigation. The bottom of the duct is fitted with an endless belt where the motion of this belt is utilized to simulate relative motion and fluid boundary-layer characteristics common to a rotating blade end.

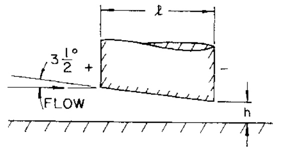


Fig. 10 Convergent gap.

In a machine where the pressure side of the blade leads in the direction of rotation (a pump or compressor), the viscous drag of the casing wall moving relative to the blade tip causes a flow, relative to the blade end, from the pressure to the suction side. This shear flow exists in addition to the leakage flow because of the pressure difference across the blade end. In a turbine, the suction side leads in rotation, and the leakage flow due to pressure drop tends to oppose the shear flow. This report contains only experimental data relating to pumps or compressors.

The investigations of tip clearance flow with the moving wall reported in this paper were carried out, using four basic gap configurations consisting of uniform, uniform with rounded corner, divergent, and convergent shapes. Clearances could be adjusted so as to vary the ratios within the range commonly found in turbomachines. The ratio of l/h ranges from 5 to 20 in standard turbomachine design, where (l) is the maximum blade thickness at the tip section. To obtain the pressure distribution across the blade end, static pressure taps were placed across the channel surface adjacent to the belt and on the upstream and downstream faces of the test specimen.

The boundary layer profile developed on the belt, when it was traveling at a speed of 83 fps, was measured by a pitot static traverse at a station 15 in. upstream of the test specimen. The boundary layer had a thickness of approximately 1 in. and the shape shown in Fig. 13. The mass flow through the tip clearance area was determined by the total pressure probe traverse at the exit plane of the tip clearance.

When considering the tip clearance flow over a rotating blade end, it must be remembered that this flow is composed of two parts: that due to the viscous drag of the casing wall moving relative to the blade tip, and that due to the pressure difference across the blade end. The mechanics of these two flows are different in that the portion of the flow contributed by the frictional drag has a very high shear rate, and consequently a much higher degree of vorticity associated with it, than with the "pressure drop" flow. It would then seem of

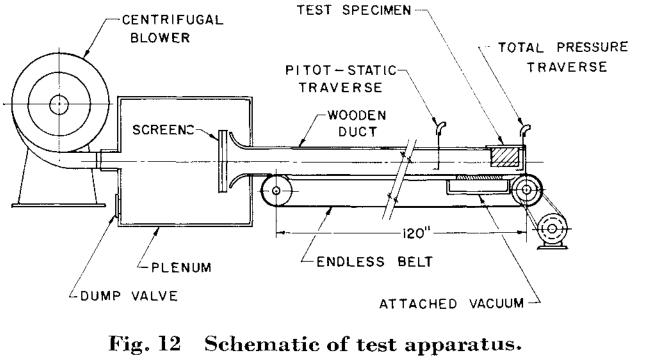


Fig. 12 Schematic of test apparatus.

interest to study the flow through the tip clearance, with the total tip clearance flow comprised of varying percentages of these two components of the flow.

To obtain such a variation, assuming other variables constant, it is necessary to vary the belt speed relative to the square root of the pressure drop across the blade end. The case of a stator blade end adjacent to a stationary wall represents a total tip-leakage flow consisting of only the "pressure drop" component.

In order to designate the relative degree of wall motion in relation to pressure drop across the blade end, a coefficient shall be defined as a blade end coefficient (E_L). This coefficient is the ratio of the pressure drop across the blade and to the dynamic pressure of the belt surface.

$$E_L = \frac{P_\infty - P_0}{\rho U^2/2} \tag{9}$$

This coefficient approaches, in magnitude, the value of a local blade lift coefficient at any chordwise point along the blade section. The local blade lift coefficient and the blade end coefficient are defined, respectively, as

$$C_L(\text{local}) = \frac{P_\infty - P_0}{\rho W^2/2} \quad \text{and} \quad E_L = \frac{P_\infty - P_0}{\rho U^2/2}$$

where P_∞ is the average pressure face pressure, P_0 is the average suction face pressure, W is the average velocity of fluid relative to blade, and U is the belt surface velocity.

To maintain blade end coefficients (E_L) comparable to those used in turbomachinery design, tests were run at a constant gap height with blade end coefficients (E_L) equal to ∞, 1.0, 0.5, 0.25, where a value of infinity results when the belt is stationary. The other values of (E_L) were obtained by maintaining a constant belt speed and varying the pressure difference across the blade end by the use of a dump valve on the plenum chamber.

Experimental Results

To obtain the flow pattern into the gap, smoke was introduced 10 in. upstream of the test specimen. Smoke also was introduced through a static pressure tap on the upstream face of the test specimens, and this smoke filament, although hard to discern in the pictures, stood out quite clearly to the naked eye and indicated any separation at the inlet area of the tip clearance.

Figure 14 illustrates the flow through a uniform gap with a sharp entrance corner; the faint smoke filament attached to the upstream corner and separating from the blade surface inside the gap illustrates the separation of the flow as it flows around the corner. This separation also is indicated by the pressure distribution through the gap, as presented later in this report.

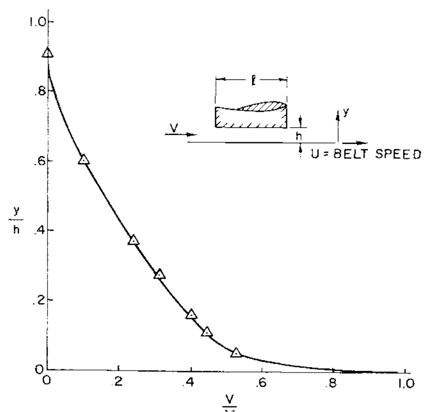


Fig. 13 Boundary-layer profile developed on moving belt.

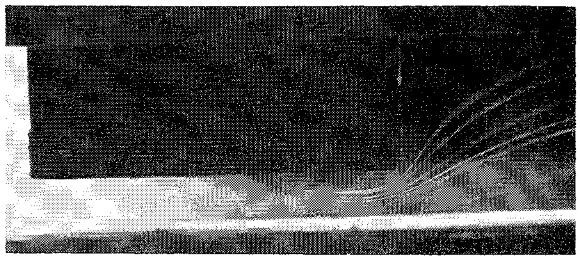


Fig. 14 Flow through a uniform gap (stationary wall).

Figure 15 illustrates the flow through a uniform gap with a rounded entrance corner, and close inspection of the figure will show a smoke filament starting on the surface of the specimen about halfway around the periphery of the corner and staying attached to the surface of the blade end throughout the gap. This indicates that separation does not occur at the tip clearance inlet for a blade end with a rounded entrance corner.

Past experience in testing shrouded propulsors has indicated that gap cavitation is dependent upon parameters such as those that follow.

1) Ratio of maximum blade tip thickness to tip clearance height (l/h).

2) Ratio of the momentum thickness (δ^*) of the boundary layer on the shroud to the tip clearance height (δ^*/h): The momentum thickness of the belt boundary layer was determined by the graphical integration of the velocity profile, measured 15 in. upstream of the test specimen. It was found to be 0.128 in. and remained relatively constant for all tests. This allowed values of δ^*/h ranging from 0.137 to 0.302 to be obtained for the various blade ends tested.

3) Ratio of the hydrodynamic tip loading to the tip speed: This parameter in a turbomachine could be expressed as the lift coefficient at the tip section, but in the test apparatus employed here is expressed as the blade end coefficient (E_L) previously defined.

4) Gap configuration (divergent, uniform, convergent, etc.): The four gap shapes tested for comparative purposes in this report were of the configuration shown in Fig. 8–11.

Figure 16 is a typical plot, illustrating the relative pressure distribution for the four gap configurations tested, where the pressure coefficient C_p shall be defined as follows:

$$C_p = (P - P_\infty)/(P_\infty - P_o) \tag{10}$$

and (P) is the pressure at any point on the blade end. This definition of C_p indicates that, as the value of C_p increases negatively, the cavitation index (σ) as previously defined in Eq. (6) increases positively.

A comprehensive comparison of the test results is shown in Fig. 17 in the form of a plot of the maximum negative pressure coefficient (C_p^*) at the gap inlet vs the blade end coefficient for the case $l/h = 14$. It will be noted that the convergent gap for all blade end coefficients (E_L) tested indicates far lower pressure coefficients than any of the other gap configurations. Another interesting characteristic is the slope of the curves for the convergent and divergent gaps. The con-

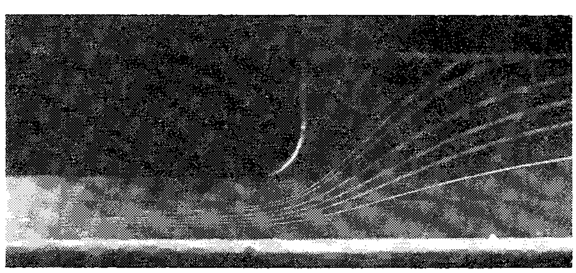


Fig. 15 Flow through a uniform gap with rounded inlet corner (stationary wall).

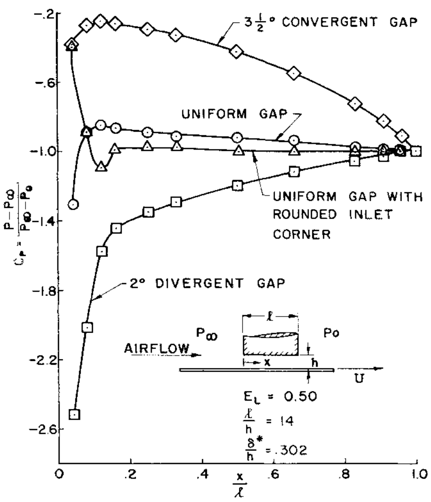


Fig. 16 Typical pressure distribution for various gap configurations.

vergent gap C_p^* decreases as the blade end coefficient decreases, whereas the divergent gap responds in exactly the opposite fashion.

Figure 18 is a plot similar to the preceding but for a lower l/h value. The maximum negative pressure coefficient at the gap inlet for all the gap configurations, with the exception of those for the divergent gap, increase with decreasing l/h. This characteristic behavior also has been noted in turbomachines where improved gap cavitation performance is obtained by decreasing the tip clearance. The slope of the curves are flatter at the smaller value of l/h, indicating that the effect of the boundary layer on the pressure distribution for varying blade end coefficients has diminished.

A series of pressure distributions for a uniform gap with varying values of blade end coefficient is illustrated in Fig. 19. The change in shape of the curves can be explained as the effect of the belt boundary layer. With $\delta^*/h = 0.302$ and $l/h = 14$ the fluid separates from the blade end. For the case where $E_L = \infty$ (belt stationary), the flow reattaches at about $x/l = 2.5$, but for the $E_L = 1.0$ and less, the effect of the vorticity in the belt boundary layer is such as to reduce the length of the separated cavity. To illustrate this effect a very large $l/h = 29$ ratio was chosen and the pressure distribution for this case is shown by the broken line in Fig. 19. It is indicated by this pressure distribution that the boundary-layer flow stagnates on the pressure face of the blade, and there is no flow around the blade end corner as occurs in the case of normal clearances.

The uniform gap configuration was used to study the effect on the pressure distribution through the tip clearance of the ratio of δ^* to gap height. Pressure distributions were ob-

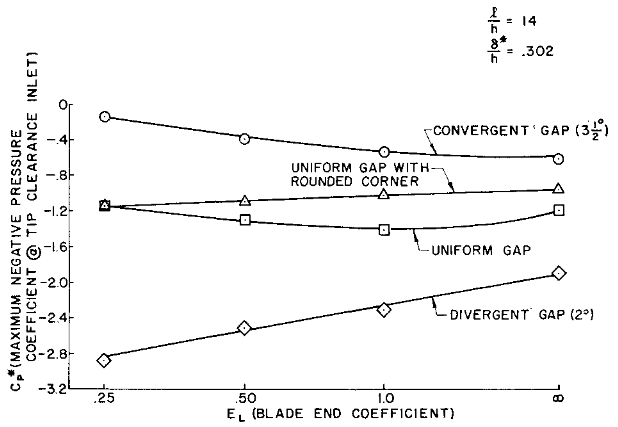


Fig. 17 C_p^* vs E_L for various gap configurations.

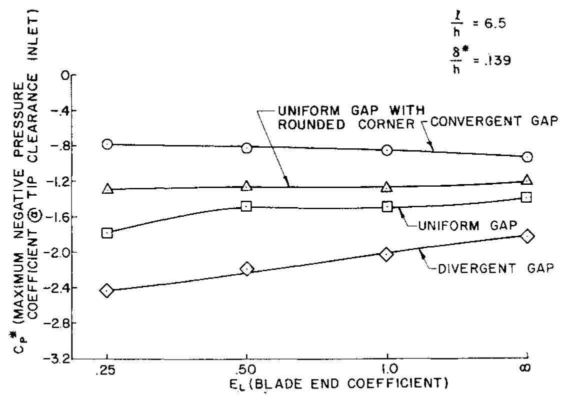


Fig. 18 C_p^* vs E_L for various gap configurations.

tained for ratios of δ^*/h equal to 0.302, 0.185, and 0.139 and the results are shown in Fig. 20.

The effects of maintaining δ^*/h constant and varying l/h can be evaluated from this plot. It is indicated that it is more effective to decrease the clearance than to increase the blade thickness, since an increase in δ^*/h produces approximately twice as much change in C_p^* as might be derived from increasing l/h by the same magnitude.

It is not a valid conclusion that the blade end $l/\delta^* = 31$ will give better cavitation performance than the thicker blade ends, as indicated by Fig. 20, because the pressure distribution for a uniform gap indicates a steep slope in the direction of increasing negative pressure coefficient at the first static pressure pickup near the inlet. Apparently the maximum negative pressure coefficient is upstream of the first static pickup and its magnitude therefore could not be measured for the various blade thicknesses. This does not mean that the slope of the curves (upon which basis the conclusion of relative effectiveness in decreasing tip clearance or increasing blade tip thickness was reached) is in error.

Figure 21 illustrates the nondimensional velocity profile through a uniform gap, as measured at the gap exit, for various blade end coefficients. A considerable increase in the nondimensional velocity is noted for decreasing values of blade end coefficient. The area enclosed by any one of these curves would represent a mass flow coefficient defined as

$$C_m = \frac{\int_0^h V dy}{[(P_\infty - P_o)/(\rho/2)]^{1/2}\, h} \qquad (11)$$

For the specific blade end coefficient considered and for the

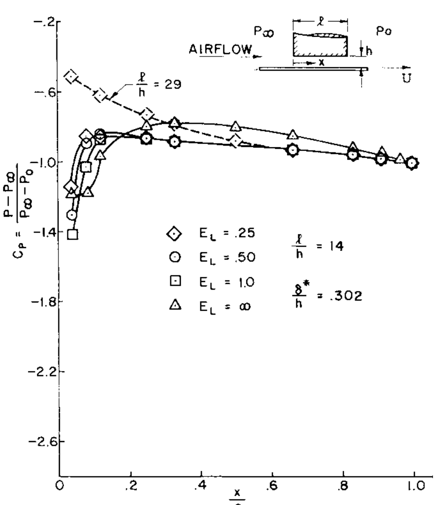


Fig. 19 Typical pressure distribution through a uniform gap for various blade end coefficients.

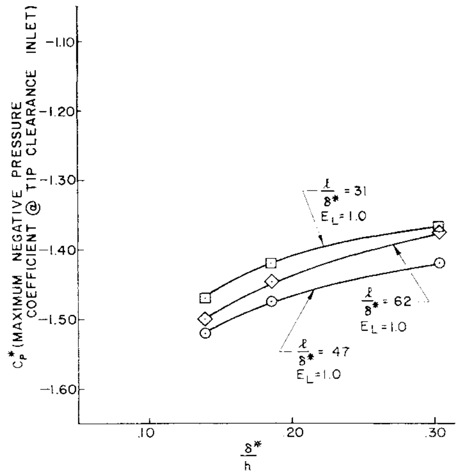


Fig. 20 C_p^* **vs** δ^*/h **for a uniform gap.**

uniform gap, an increase in the mass flow coefficient of approximately 45% is experienced when the blade end coefficient is decreased from infinite to 0.25.

The nondimensional velocity profiles, again measured at the channel exit, for various gap configurations are illustrated in Fig. 22. As might be expected, the divergent gap has a velocity profile of smaller magnitude than the other gap shapes indicating that this gap configuration would give better performance in respect to tip vortex cavitation. This conclusion is drawn based on the previous assumption that the strength of the vortex sheet (formed by the interaction of the tip clearance flow and the main blade passage flow on the suction face of the blade) is directly dependent upon the exit velocity of the tip clearance flow. The convergent gap has the velocity profile of greatest magnitude, thereby indicating poor performance associated with tip vortex cavitation.

There are not specific test data concerning the relative effects on the tip vortex cavitation, when using either a convergent or a divergent gap, but past use of uniform gaps with rounded corners and also convergent gaps in propulsor design and testing at the Garfield Thomas Water Tunnel have not created any significant changes in tip vortex cavitation; they have provided improvements in gap cavitation by a factor of 2.

A comprehensive comparison on mass flow associated with the various gap configurations is illustrated in Fig. 23. It is also of interest to point out that mass flow coefficients of 0.92 for the uniform gap with rounded corner and 0.82 for the uniform gap, when considering $E_L = \infty$, compared to within 1% of Shalnev's experimentally determined values, using water as the test medium.

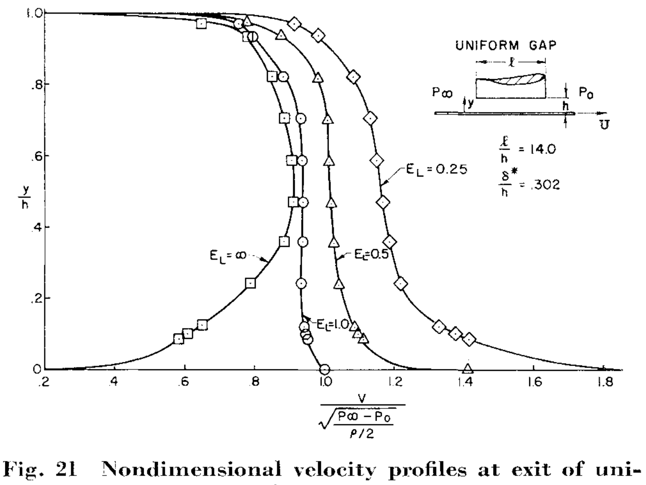


Fig. 21 Nondimensional velocity profiles at exit of uniform gap.

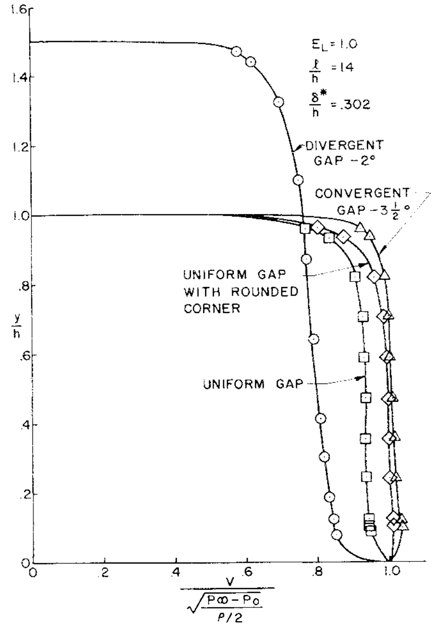


Fig. 22 Nondimensional velocity profiles for various gap configurations.

Conclusions and Recommendations

The experimental data presented in this report indicate that a convergent gap configuration will provide the best gap cavitation performance in comparison to the other configurations tested. The magnitude of the mean velocity of the tip clearance flow associated with a convergent gap is greater than that of the other configurations. Preliminary analyses indicate that this characteristic may promote the occurrence of tip vortex cavitation although no specific tests have been run to confirm this. It was found that a very slightly divergent gap configuration provided very poor gap cavitation performance. The mass flow associated with the divergent gap is greater than that of the other configurations; however, the magnitude of its mean velocity at the gap exit is smaller than any of the other configurations. This would indicate good tip vortex cavitation performance but larger losses in over-all machine efficiency.

The effect of the relative motion of the blade end to a wall is to increase the occurrence of cavitation in all of the gap configurations tested, except the convergent gap, which demonstrated an opposite effect. The tip clearance mass flow was increased in all cases because of the effect of relative motion between casing wall and blade end.

Attempts to obtain an exact analytical solution of the flow through the gap are shown in Ref. 9 and require a consideration of both inertia and viscous terms in the solution of the

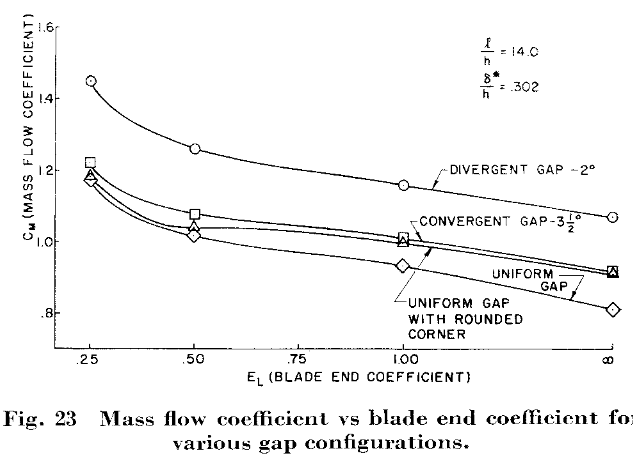


Fig. 23 Mass flow coefficient vs blade end coefficient for various gap configurations.

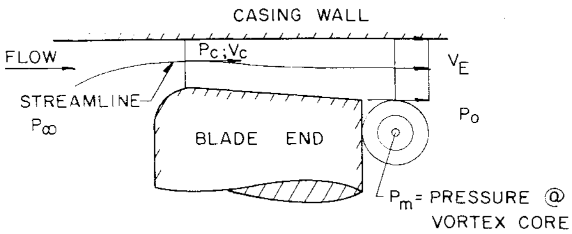


Fig. 24 Divergent gap with rounded inlet.

Navier Stokes equations. The difficulties encountered in obtaining an exact solution to these equations indicate that a semiempirical solution may well be the most effective means of predicting gap cavitation performance in turbomachines.

In the previous discussion, the tip clearance flow has been idealized and simplified in order to draw qualitative conclusions on the mechanics of the flow and its associated cavitation phenomenon. It must be remembered that, for the actual blade end problem in a turbomachine, the pressure distribution over the blade tip section undoubtedly will be altered from that for which it was originally designed, because it is operating in the boundary layer of the casing wall. This will tend to load the blade tip section more than originally planned and the maximum negative pressure on the suction face of the blade will not only increase, but will also move nearer the leading edge of the blade. It might be postulated that this local low-pressure area on the suction surface of the blade, coupled with the low pressure associated with the tip vortex, initiates the tip vortex cavitation.

It also might be pointed out that the blade thickness near the leading edge is usually quite small in order to avoid local low-pressure regions required for good cavitation performance, and this would complicate the use of the suggested blade end configurations. To alleviate this problem, trailing edge loaded blades are suggested with the tip sections of the blade designed to accommodate the casing wal lboundary layer. This would move the location of the maximum negative pressure on the suction face of the blade aft and also reduce its magnitude. It also would place the maximum blade thickness in the same area as the maximum pressure difference, thereby placing at the disposal of the designer some finite blade end thickness, which he might shape as already suggested.

An optimum gap configuration can be suggested when it is considered that rounding the corner of the blade on the pressure face provides good resistance to "gap cavitation." It also was deduced that the tip vortex form of cavitation was a function of the velocity of the tip clearance flow as it issued from the exit of the gap, and when the diffusing characteristics of a divergent gap are considered, it is apparent that the divergent gap is attractive from the standpoint of tip vortex cavitation. It then would appear that the combination of these two shapes into a configuration as illustrated in Fig. 24 would provide better over-all cavitation resistance than any of the models tested in this report. The major disadvantage is that the quantity of tip leakage flow associated with this configuration is in the order of $1\frac{1}{2}$ to 2 times as great as that associated with a uniform gap having a rounded entrance corner and would possibly be detrimental to the efficiency of the machine.

An approximation of an optimum gap configuration, shaped as previously described, can be obtained as follows (referring to Fig. 24 where all velocities are averaged based on mass flow). For optimum performance, the following must be true:

$$P_c = P_m = P_v \tag{12}$$

Along a streamline, considering no losses,

$$P_c = P_T - \rho(V_c^2/2) \tag{13}$$

Assuming solid body rotation of the vortex, the pressure at the center is

$$P_m = P_T - \rho V_E^2 \tag{14}$$

Therefore

$$V_c = (2V_E)^{1/2}$$

The preceding indicates that the height at the exit area of the gap should be 1.414 times as large as the height at the constricted area.

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